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SOME ANALOGUES OF TOPOLOGICAL GROUPS

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Abstract

Let $(G, *)$ be a group and τ be a topology on G. Let $\tau^\alpha = \{A \subseteq G : A \subseteq Int(Cl(Int(A)))\}$, $g * \tau = \{g * A : A \in \tau\}$ for $g \in G$. In this paper, we establish two relations between G and τ under which it follows that $g * \tau \subseteq \tau^\alpha$ and $g * \tau^\alpha \subseteq \tau^\alpha$, designate them by α-topological groups and α-irresolute topological groups, respectively. We indicate that under what conditions an α-topological group is topological group. This paper also covers some general properties and characterizations of α-topological groups and α-irresolute topological groups. In particular, we prove that (1) the product of two α-topological groups is α-topological group, (2) if H is a subgroup of an α-irresolute topological group, then $\alpha Int(H)$ is also subgroup, and (3) if A is an α-open subset of an α-irresolute topological group, then $< A >$ is also $\alpha-$open. In the mid of discourse, we also mention about their relationships with some existing spaces.

Keywords: α-open sets, α-closed sets, α-topological groups, α-irresolute topological group.

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1. Introduction and preliminaries

The study of topological groups is quite well-known in mathematics and prominent in many areas of mathematics. Assuming familiarity with the features and applications of topological groups, we recall some similar notions and generalizations of topological groups: Semi-topological groups [9, 10], s-topological groups [1, 2], S-topological groups [1], Quasi S-topological groups [4], Irresolute topological groups [3, 8], Quasi-irresolute topological groups [11] and paratopological groups [13]. Also, we obtained a new useful generalization of topological groups in [12].

These structures are mainly developed by investigating the group structure via some weaker or stronger forms of continuity. This kind of investigation is quite useful and successful for the development of new notions having significance analogous to topological groups.

In this paper, we introduce two new structures of topology and group theory which are resembling to topological groups. This paper also involves some important properties and characterizations of α-topological groups and α-irresolute topological groups. We comprehend these concepts through some examples and counter examples which also speak volumes about their relationship. We will observe that the class of α-topological groups is a fine generalization of topological groups. Concerning the obverse of the fact, we point out some conditions with which an α-topological group is topological group.

In 1965, Njastad [6] introduced a weaker but useful class of open sets, the class of α-open sets, in topological spaces. He defines a set A in a topological space X to be α-open if $A \subseteq Int(Cl(Int(A)))$. The complement of an α-open set is said to be α-closed set; or equivalently, a set A in a topological space X is α-closed if $Cl(Int(Cl(A))) \subseteq A$.

For a subset A of X, the α-closure of A is the intersection of all α-closed sets in X containing A and the α-interior of A is the union of all α-open sets in X that are contained in A. These are denoted by $\alpha Cl(A)$ and $\alpha Int(A)$, respectively. It is known that a subset A of X is α-closed if and only if $A = \alpha Cl(A)$. Moreover, a point $x \in \alpha Cl(A)$ if and only if $A \cap U \neq \emptyset$ for each α-open set U in X containing x. A subset A of X is α-open if and only if $A = \alpha Int(A)$. A point x of X is called an α-interior point of a set A in X if there exists an α-open set U in X containing x such that $U \subseteq A$. The set of all α-interior points of A is equal to $\alpha Int(A)$. The family of all α-open (resp. α-closed) sets in X will be denoted by τ^α (resp. $\alpha C(X)$) where τ denotes the topology of X. Njastad [6] showed that τ^α also forms a topology on X. For $x \in X$, $\tau^{(\alpha,x)}$ denotes the collection of all α-open sets in X containing x.

2. α-topological groups

This section acquaints us with the notion and some examples of α-topological groups. We will see that every topological group is an α-topological group but the converse is not true in general. In this connection, we prove a theorem, Theorem 2.1, which tells us under what conditions the converse holds. We also establish some basic properties and characterizations of α-topological groups.

By G, we mean the group $(G, *)$ where '$*$' is a binary operation on G under which G is a group. For $A, B \subseteq G$, let $A * B = \{a * b : a \in A,\ b \in B\}$ and $A^{-1} = \{a^{-1} : a \in A\}$. For $g, h \in G$, we write $g * B$ or $A * h$ rather than $\{g\} * B$ or $A * \{h\}$. The identity element of G is denoted by e.

Definition 2.1. An α-topological group, denoted by (G, τ), is a group G endowed with a topology τ such that.

(1) For each $x, y \in G$ and each open set W in G containing $x * y$, there exist α-open neighborhoods U and V of x and y, respectively, in G such that $U * V \subseteq W$, and

(2) For each $x \in G$ and each open set V in G containing x^{-1}, there exists an α-open neighborhood U of x in G such that $U^{-1} \subseteq V$.

Example 2.1. Let H be the additive group of real numbers. Then H with its usual topology τ, is an α-topological group.

Example 2.2. Let $(G, *)$ be a group of order 4. Let A, B be two subsets of G such that

(i) $G = A \cup B$ and $A \cap B = \emptyset$.

(ii) $g * A = A$ or B and $g * B = A$ or B for each $g \in G$.

(iii) $A * A = A$, $B * B = A$ and $A * B = B$.

Let $\tau = \{\emptyset, A, B, G\}$ be the topology on G. Then (G, τ) is an α-topological group.

Proof. Let $G = \{e, a, b, c\}$ where e denotes the identity element in G. By conditions (i) and (iii) of the hypothesis, $A \neq \emptyset$ and $B \neq \emptyset$. Notice that G has an element $x \neq e$ such that $x^2 = e$. Let a be the such element. By Condition (iii) with previous argument, we must have

$$A = \{e, a\} \text{ and } B = \{b, c\}.$$

Observe that

$$A^{-1} = A \text{ and } B^{-1} = B.$$

Now, let $x, y \in G$ and W be an open neighborhood of $x * y$ in G. If $W = A$, we have either both $x, y \in A$ or both $x, y \in B$. Consequently, owing to Condition (iii) of hypothesis, we obtain α-open sets U and V in G containing x and y respectively such that $U * V \subseteq A$. If $W = B$, then we see that either $x \in A$, $y \in B$ or $y \in A$, $x \in B$. Again, by Condition (iii) of hypothesis, we find out α-open neighborhoods U of x and V of y such that $U * V \subseteq B$. Finally, for any $x \in G$ and any open neighborhood V of x^{-1}, we can choose α-open neighborhood $U = V$ of x such that $U^{-1} \subseteq V$. Hence (G, τ) is α-topological group. ■

It is obvious from the definition that every α-topological group is s-topological group [1], but the converse is not true in general as below example shows.

Example 2.3. Consider the additive group $(\mathbb{R}, +)$ of real numbers endowed with the topology τ generated by the family of sets $\{(a,b) : a,b \in \mathbb{R}\} \cup \{[c,d) : c,d \in \mathbb{R}, 0 < c < d\}$. Then, $(\mathbb{R}, \tau)$ is an s-topological group but it is not α-topological group.

By Definition 2.1, it follows immediately that every topological group is an α-topological group, but the converse is not true, in general. We give a theorem about the converse of this fact.

Theorem 2.1. *Let $(G, \wp)$ be a regular α-topological group such that every nowhere dense subset of G is closed. Then $(G, \wp)$ is topological group if and only if $\wp = \wp^{\alpha}$.*

Proof. One way is trial. Conversely, let $(G, \wp)$ be a topological group and A be an α-open subset of G. Then $A = B - N$ where B is open subset of G and N is nowhere dense subset of G.

Claim. *A is open.*

Let x be any element of A. By hypothesis, N is closed, and thus, there exist open subsets U and V of G such that $x \in U$, $N \subseteq V$ and $U \cap V = \emptyset$. This implies that there exists an open subset P of G such that $x \in P \subseteq A$. Hence the claim. ■

Definition 2.2. Let X and Y be two topological spaces. A function $f : X \to Y$ is α-continuous [5] if $f^{-1}(U)$ is α-open set in X, for every open set U of Y.

Theorem 2.2. *Let G be an α-topological group and $g \in G$. Then*

(1) *the mapping $h_g : G \to G$ defined by $h_g(x) = g * x$, $\forall x \in G$, is α-continuous,*
(2) *the mapping $l_g : G \to G$ defined by $l_g(x) = x * g$, $\forall x \in G$, is α-continuous.*

Proof. Straightforward. ■

Theorem 2.3. *Let N be an open set in an α-topological group G with $e \in N$. Then, the following implications hold:*

(1) *There exists $U \in \tau^{(\alpha,e)}$ such that $U * U \subseteq N$.*
(2) *There exists $U \in \tau^{(\alpha,e)}$ such that $U^{-1} \subseteq N$.*

Proof. A simple. ■

Theorem 2.4 (Necessary Condition)**.** *Let A be any open set in an α-topological group G. The following are valid:*

(1) *$g * A \in \tau^{\alpha}$ for all $g \in G$.*

(2) $A * g \in \tau^{\alpha}$ *for all* $g \in G$.

(3) $A^{-1} \in \tau^{\alpha}$.

Proof. We only prove (1) and (3). The proof for part (2) follows along similar lines.

(1) For $a \in A$, let $x = g*a$. By Theorem 2.2, $h_{g^{-1}}$ is α-continuous. Therefore,

$$\exists\, U \in \tau^{(\alpha,x)} \text{ such that } h_{g^{-1}}(U) \subseteq A.$$

That is, $g^{-1} * U \subseteq A$.

This implies that $U \subseteq g * A$. Thus, $x \in \alpha Int(g * A)$ showing that $g * A$ is α-open.

(3) Let $y = x^{-1} \in A^{-1}$ where x is an element of A. By Definition 2.1, we get an α-open set U in G containing y such that $U^{-1} \subseteq A \Rightarrow U \subseteq A^{-1} \Rightarrow y \in \alpha Int(A^{-1})$. Hence the assertion follows. ■

Using Theorem 2.4, it can be easily checked that Sorgenfrey line is not an α-topological group. Explicitly,

Example 2.4. Consider the additive group G of reals. Let $\mathcal{L}$ be the lower-limit topology on $G = \mathbb{R}$. Then $(G, \mathcal{L})$ is not an α-topological group because for open set $A = [1, 2)$, $-A = (-2, -1]$ is not α-open in G.

Corollary 2.4.1. *Let B be any closed set in an α-topological group G. Then*

(1) $g * B \in \alpha C(G)$ *for each* $g \in G$.

(2) $B^{-1} \in \alpha C(G)$.

Theorem 2.5 (Characterizations). *Let G be an α-topological group. For $A \subseteq G$, the following hold:*

(1) $\alpha Cl(g * A) \subseteq g * Cl(A)$ *for each* $g \in G$.

(2) $g * \alpha Cl(A) \subseteq Cl(g * A)$ *for each* $g \in G$.

(3) $g * Int(A) \subseteq \alpha Int(g * A)$ *for each* $g \in G$.

(4) $Int(g * A) \subseteq g * \alpha Int(A)$ *for each* $g \in G$.

Proof. (1) For $x \in \alpha Cl(gA)$, set $y = g^{-1} * x$. Let W be any open set in G containing y. Then

$$\exists\, U \in \tau^{(\alpha,\, g^{-1})} \text{ and } V \in \tau^{(\alpha,x)} \text{ such that } U * V \subseteq W.$$

Since $x \in \alpha Cl(gA)$, there is $a \in gA \cap V$.

This implies that $g^{-1} * a \in A \cap U * V \subseteq A \cap W \Rightarrow A \cap W \neq \emptyset$. Hence $y \in Cl(A)$; that is, $x \in gCl(A)$.

(2) For $x \in \alpha Cl(A)$, let W be an open neighborhood of $g * x$. Then

$$\exists\, U \in \tau^{(\alpha,\, g)} \text{ and } V \in \tau^{(\alpha,x)} \text{ satisfying } U * V \subseteq W.$$

Since $x \in \alpha Cl(A)$, $A \cap V \neq \emptyset$. This results in $g * A \cap W \neq \emptyset$, i.e., $g * \alpha Cl(A) \subseteq Cl(g * A)$.

(3) A direct consequence of Theorem 2.4.

(4) Pick an arbitrary point $x \in Int(g * A)$. Then $x = g * y$ for some $y \in A$. There exist $U \in \tau^{(\alpha,g)}$ and $V \in \tau^{(\alpha,y)}$ such that

$$U * V \subseteq g * A.$$

In particular, $g*V \subseteq g*A$. Thus, $g*V \subseteq g*\alpha Int(A)$. Whereby the assertion follows. ■

Under certain extra condition on A in Theorem 2.5, we obtain the following result.

Theorem 2.6. *Let A be an open set in an α-topological group G. Then $Cl(g * A) = g * Cl(A)$ for each $g \in G$.*

To prove Theorem 2.6, we use the following result.

Lemma 2.7 [5, Theorem 1.1]**.** *Let $f : X \to Y$ be a mapping, then the following statements are equivalent.*

(1) *f is α-continuous.*

(ii) *$f[Cl(Int(Cl(A)))] \subseteq Cl(f(A))$ for each $A \subseteq X$.*

***Proof of Theorem* 2.6.** Since A is open, $Cl(A) = Cl(Int(Cl(A)))$. By Theorem 2.2, h_g is α-continuous. Now by above lemma, we obtain that

$$g * Cl(Int(Cl(A))) \subseteq Cl(g * A).$$

This gives,

$$g * Cl(A) \subseteq Cl(g * A).$$

Next, it follows from Corollary 2.4.1 that $Cl(g * A) \subseteq Cl(Int(Cl(g * A))) \subseteq g * Cl(A)$. This finishes the proof.

Theorem 2.8. *Let A and B be any subsets of an α-topological group G. Then $\alpha Cl(A) * \alpha Cl(B) \subseteq Cl(A * B)$.*

Proof. Straightforward. ■

Theorem 2.9. *Under the same conditions of Theorem* 2.5*, the following hold:*

(1) $\alpha Cl(A^{-1}) \subseteq [Cl(A)]^{-1}$.

(2) $[\alpha Cl(A)]^{-1} \subseteq Cl(A^{-1})$.

(3) $[Int(A)]^{-1} \subseteq \alpha Int(A^{-1})$.

(4) $Int(A^{-1}) \subseteq [\alpha Int(A)]^{-1}$.

Theorem 2.10. *The product of two α-topological groups is α-topological group.*

Proof. Let $(G, *_1, \tau_1)$ and $(H, *_2, \tau_2)$ be two α-topological groups. We show that $(G\times H, *, \tau)$ is an α-topological group, where τ is the product topology on $G\times H$ and '$*$' is a binary operation on $G \times H$, defined as

$$(x, y) * (x', y') = (x *_1 x', y *_2 y') \text{ for } (x, y), (x', y') \in G \times H.$$

Pick arbitrary points $\zeta = (x, y)$ and $\eta = (g, h)$ from $G \times H$ and let W be an open neighborhood of $\zeta * \eta$. Then there exist open sets A in G containing $x *_1 g$ and B in H containing $y *_2 h$. There exist $U_1 \in \tau_1^{(\alpha,x)}$, $U_2 \in \tau_2^{(\alpha,y)}$, $V_1 \in \tau_1^{(\alpha,\ g)}$ and $V_2 \in \tau_2^{(\alpha,h)}$ such that $U_1 *_1 V_1 \subseteq A$ and $U_2 *_2 V_2 \subseteq B$. Hence we get α-open sets $U = U_1 \times U_2$ and $V = V_1 \times V_2$ containing ζ and η, respectively such that $U * V \subseteq W$.

Next, for $\zeta = (x,\ y) \in G \times H$, let W be an open neighborhood of ζ^{-1}. Then $W = U \times V$ where U is open subset of G containing x^{-1} and V is open subset of H containing y^{-1}. As a result of this, there exist α-open sets $P \in \tau_1^{(\alpha,x)}$ and $Q \in \tau_2^{(\alpha,y)}$ such that $P^{-1} \subseteq U$ and $Q^{-1} \subseteq V$. Thus, $P\times Q$ is α-open set in $G\times H$ with $(x, y) \in P \times Q$ and $[P \times Q]^{-1} \subseteq W$. This ends the proof. ■

3. α-irresolute topological groups

This section deals with the α-irresolute topological groups and several indispensable features of them. In particular, we prove that (1) if H is a subgroup of an α-irresolute topological group G, then $\alpha Int(H)$ is also subgroup of G and (2) if A is any non-empty α-open subset of G, then $< A >$ is also α-open.

Definition 3.1. An α-irresolute topological group (G, τ) is a group G endowed with a topology τ such that

(1) For each $x, y \in G$ and each α-open set W in G containing $x * y$, there exist α-open sets U and V in G containing x and y, respectively such that $U * V \subseteq W$, and

(2) For each $x \in G$ and each α-open set V in G containing x^{-1}, there exists an α-open set U in G containing x such that $U^{-1} \subseteq V$.

Example 3.1. Consider the multiplicative group $\mathbb{Z}_5 = \{1, 2, 3, 4\}$ with the topology τ on $\mathbb{Z}_5$ where $\tau = \{\emptyset, \{1, 4\}, \{2, 3\}, \mathbb{Z}_5\}$. Then $(\mathbb{Z}_5, \tau)$ is an α-irresolute topological group.

Theorem 3.1. *Let (G, τ) be an α-irresolute topological group. For any $A \in \tau^\alpha$, the following hold:*

(1) $g * A \in \tau^\alpha$ *for each* $g \in G$.

(2) $A * g \in \tau^\alpha$ *for each* $g \in G$.

(3) $A^{-1} \in \tau^\alpha$.

Proof. (1) Let $x \in g * A$ be any element. Then $\exists\ U \in \tau^{(\alpha, g^{-1})}$ and $V \in \tau^{(\alpha, x)}$ such that

$$U * V \subseteq A$$

That is, $V \subseteq g * A$. Hence $x \in \alpha Int(g * A)$. Therefore, $g * A$ is α-open.

(2) Follows along similar lines as above.

(3) Let $y = x^{-1}$ be an arbitrary point of A^{-1} where x is an element of A. By Definition 3.1,

$$\exists\ U \in \tau^{(\alpha,\ y)} \text{ such that } U \subseteq A^{-1}.$$

That is, $y \in \alpha Int(A^{-1})$. Thus, A^{-1} is α-open. ■

The following result about α-irresolute topological groups is elementary but indispensable. The proof of this result is petty and thus, not given

Theorem 3.2. *Let G be an α-irresolute topological group. For $V \in \tau^{(\alpha,e)}$, the following assertions hold:*

(1) *There exists* $U \in \tau^{(\alpha,e)}$ *such that* $U * U \subseteq V$.

(2) *There exists* $U \in \tau^{(\alpha,e)}$ *such that* $U^{-1} \subseteq V$.

(3) *There exists a symmetric* $U \in \tau^{(\alpha,e)}$ *(i.e.,* $U = U^{-1}$*) such that* $U * U \subseteq V$.

(4) *There exists a symmetric* $U \in \tau^{(\alpha,e)}$ *such that* $U^{-1} \subseteq V$.

(5) *For each* $g \in G$, *there exists* $U \in \tau^{(\alpha,e)}$ *such that* $g^{-1} * U * g \subseteq V$.

Proof. Simple and therefore, omitted. ■

Theorem 3.3. *Let G be an α-irresolute topological group. For any $A \subseteq G$, the following assertions hold:*

(1) $\alpha Cl(g * A) = g * \alpha Cl(A)$ *for all* $g \in G$.

(3) $\alpha Cl(A^{-1}) = [\alpha Cl(A)]^{-1}$.

Proof. (1) For $x \in \alpha Cl(g * A)$, set $y = g^{-1} * x$. If $W \in \tau^{(\alpha,\ y)}$, there exists $U \in \tau^{(\alpha,\ g^{-1})}$ and $V \in \tau^{(\alpha,\ x)}$ such that

$$U * V \subseteq W.$$

In particular,

$$g^{-1} * V \subseteq W.$$

Since $x \in \alpha Cl(g * A)$, $g * A \cap V \neq \emptyset$. This implies that $g * A \cap g * W \neq \emptyset$. Consequently, $A \cap W \neq \emptyset$. Thereby it follows that $\alpha Cl(g * A) \subseteq g * \alpha Cl(A)$. On the other hand, for any $x \in \alpha Cl(A)$ and any α-open neighborhood W of $g * x$, we have

$$U * V \subseteq W \text{ for some } U \in \tau^{(\alpha,g)}, \text{ and } V \in \tau^{(\alpha,x)}.$$

Thence we find that $A \cap V \neq \emptyset$ and as a result, $g * A \cap W \neq \emptyset$. Therefore, $g * \alpha Cl(A) \subseteq \alpha Cl(g * A)$. This completes the proof of part (1).

(2) Let x be any point of $\alpha Cl(A^{-1})$ and $V \in \tau^{(\alpha,x^{-1})}$. There exists $U \in \tau^{(\alpha,\ x)}$ with $U^{-1} \subseteq V$. By assumption, $A^{-1} \cap U \neq \emptyset$. This gives that $A \cap U^{-1} \neq \emptyset \Rightarrow A \cap V \neq \emptyset$. Thus, $x \in [\alpha Cl(A)]^{-1}$. Conversely, pick an arbitrary point y of $[\alpha Cl(A)]^{-1}$ and let V be an α-open set in G containing y. Then $y = x^{-1}$ for some $x \in \alpha Cl(A)$ and there exists $U \in \tau^{(\alpha,\ x)}$ such that $U^{-1} \subseteq V$. Now,

Since $x \in \alpha Cl(A)$, $A \cap U \neq \emptyset$. This results in $A^{-1} \cap U^{-1} \neq \emptyset \Rightarrow A^{-1} \cap V \neq \emptyset$. Therefore, $y \in \alpha Cl(A^{-1})$. Hence the assertion follows. ■

Theorem 3.4. *Under the same statement as Theorem 3.3, the following hold:*

(1) $\alpha Int(g * A) = g * \alpha Int(A)$ *for all* $g \in G$.

(2) $\alpha Int(A^{-1}) = [\alpha Int(A)]^{-1}$.

Proof. (1) It follows immediately from Theorem 3.1 that $g * \alpha Int(A) \subseteq \alpha Int(g * A)$. On the obverse side, let x be an arbitrary point of $\alpha Int(g * A)$. Then for some $a \in A$, there exist $U \in \tau^{(\alpha,g)}, V \in \tau^{(\alpha,a)}$ with $U * V \subseteq g * A$. Whence we have that $x \in g * \alpha Int(A)$. Thus, $\alpha Int(g * A) = g * \alpha Int(A)$.

(2) By Theorem 3.1, $[\alpha Int(A)]^{-1} \subseteq \alpha Int(A^{-1})$. Next, for $x \in \alpha Int(A^{-1})$, there exists an α-open set U in G containing h such that $U^{-1} \subseteq A^{-1}$ for some $h \in A$. From this fact, we get that $\alpha Int(A^{-1}) \subseteq [\alpha Int(A)]^{-1}$. Hence the proof. ■

We recall a fact from group theory that if A is a subset of G, then $< A >$ denotes the intersection of all subgroups of G containing A. Indeed, $< A >$ is the smallest subgroup of G containing A. Symbolically,

$$< A >= \bigcap \{H \subset G : A \subseteq H,\ H \leq G\},$$

where $H \leq G$ denotes that H is subgroup of G.

Theorem 3.5. *Let* G *be an* α*-irresolute topological group.*

(1) *If* H *is a subgroup of* G*, then* $\alpha Int(H)$ *is also subgroup of* G*, provided* $\alpha Int(H) \neq \emptyset$.

(2) *If $A \subseteq G$ is a non-empty α-open set, then $< A >$ is α-open.*

Proof. (1) Follows immediately from Theorem 3.1.

(2) By definition,

$$< A >= \bigcap \{H : A \subseteq H,\ H \leq G\}.$$

Since A is α-open, $< A >= \bigcap\{\alpha Int(H) : A \subseteq H,\ H \leq G\}$. By a result in [6], it follows that $< A >$ is α-open. ■

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